THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

MAT2200 ASSIGNMENT 1-May 6, 2020

DUE DATE: 18/05/2020

SYSTEMS OF LINEAR EQUATIONS-Prepared by Dr. C. CHILESHE

INSTRUCTIONS:

- This assignment comprises nine(9) questions, you are advised to attempt all the questions.
- Submit your hand written solutions **only** on the **moodle platform**.

1. Determine the pivot and free variables in each of the following systems and solve the systems:

   $(a)$

$$
\begin{aligned}
2x_1 - 3x_2 - 6x_3 - 5x_4 + 2x_5 &= 7 \\
x_3 + 3x_4 - 7x_5 &= 6 \\
x_4 - 2x_5 &= 1.
\end{aligned}
$$

   $(b)$

$$
\begin{aligned}
x + 2y - 3z + 2t &= 2 \\
2x + 5y - 8z + 6t &= 5 \\
3x + 4y - 5z + 2t &= 4.
\end{aligned}
$$

   $(c)$

$$
\begin{aligned}
x - 2y &= 5 \\
2x + 3y &= 3 \\
3x + 2y &= 7.
\end{aligned}
$$

2. In each of the following systems, determine:

   $(i)$ the values of $a$ such that the system has a unique solution,

   $(ii)$ the pairs of values of $(a, b)$ so that the system has more than one solution,

   $(iii)$ why the value of $b$ does not have an effect on whether the system has a unique solution.

$(a)$

$$\begin{aligned} x - 2y &= 1 \\ x - y + az &= 2 \\ ay + 9z &= b. \end{aligned}$$

$(b)$

$$\begin{aligned} x + 2y + 2z &= 1 \\ x + ay + 3z &= 3 \\ x + 11y + az &= b. \end{aligned}$$

$(c)$

$$\begin{aligned} x + y + az &= 1 \\ x + ay + z &= 4 \\ ax + y + z &= b. \end{aligned}$$

3. Write $v$ as a linear combination of $u_1, u_2, u_3$:

   $(a)$ $v = (4, -9, 2),\ u_1 = (1, 2, -1),\ u_2 = (1, 4, 2),\ u_3 = (1, -3, 2)$,

   $(b)$ $v = (1, 3, 2),\ u_1 = (1, 2, 1),\ u_2 = (2, 6, 5),\ u_3 = (1, 7, 8)$,

   $(c)$ $v = (1, 4, 6),\ u_1 = (1, 1, 2),\ u_2 = (2, 3, 5),\ u_3 = (3, 5, 8)$.

4. Let $u_1 = (1, 1, 2)$, $u_2 = (1, 3, -2)$, $u_3 = (4, -2, -1)$ in $\mathbb{R}^3$. Show that $u_1,\ u_2$ and $u_3$ are orthogonal and write $v$ as a linear combination of $u_1,\ u_2$ and $u_3$ where

   $(a)$ $v = (1, -3, 3)$,

   $(b)$ $v = (1, 1, 1)$,

   $(c)$ $v = (5, -5, 9)$.

5. Find the dimension and a basis of the general solution $W$ of each of the following systems:

   $(a)$

$$\begin{aligned} x_1 + 3x_2 + 2x_3 - x_4 - x_5 &= 0 \\ 2x_1 + 6x_2 + 5x_3 + x_4 - x_5 &= 0 \\ 5x_1 + 15x_2 + 12x_3 + x_4 - 3x_5 &= 0. \end{aligned}$$

(b)

$$
\begin{aligned}
2x_1 - 4x_2 + 3x_3 - x_4 + 2x_5 &= 0\\
3x_1 - 6x_2 + 5x_3 - 2x_4 + 4x_5 &= 0\\
5x_1 - 10x_2 + 7x_3 - 3x_4 + 18x_5 &= 0.
\end{aligned}
$$

6. Reduce each of the following matrices in:

   (i) echelon form,

   (ii) row canonical form.

   (a)
$$
\begin{bmatrix} 2 & 4 & 2 & -2 & 5 & 1\\ 3 & 6 & 2 & 2 & 0 & 4\\ 4 & 8 & 2 & 6 & -5 & 7 \end{bmatrix},
$$

   (b)
$$
\begin{bmatrix} 0 & 1 & 2 & 3\\ 0 & 3 & 8 & 12\\ 0 & 0 & 4 & 6\\ 0 & 2 & 7 & 10 \end{bmatrix},
$$

   (c)
$$
\begin{bmatrix} 1 & 3 & 1 & 3\\ 2 & 8 & 5 & 10\\ 1 & 7 & 7 & 11\\ 3 & 11 & 7 & 15 \end{bmatrix}.
$$

7. Using only $0's$ and $1's$, find the number $n$ of possible $3 \times 3$ matrices in row canonical form.

8. Prove that delelting the last column of an echelon form (respectively, the row canonical form) of an augmented matrix $M = [A, B]$ yields an echelon (respectively, the row canonical form) of $A$.

9. Matrix $A$ is equivalent to matrix $B$, written $A \approx B$, if there exist nonsingular matrices $P$ and $Q$ such that $B = PAQ$. Prove that $\approx$ is an equivalence relation.