8.2. Principle of Mathematical Induction

Exercises

Use Principle of Mathematical Induction and prove that

a) $1^3+2^3+3^3+\ldots+n^3=\left(\frac{n(n+1)}{2}\right)^2$ for all $n\in N$

b) $5^n+2.(11^n)$ is a multiple of 3 for all $n\in N.$

c) $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\left\{1-\frac{1}{(n+1)^2}\right\}=\frac{n+2}{2n+2}$ for all $n\in N$.

d) $n^7-7n^5+14n^3-8n$ is divisible by 840 for all $n\in N$.

e) $2^2+(2^2+4^2)+(2^2+4^2+6^2)+\ldots n\, terms=\frac{n(n+1)^2(n+2)}{2}$ for all $n\in N$

f) $1.3+2.3^2+3.3^3+\ldots+n.3^n=\frac{(2n-1)3^{n+1}+3}{4}$ for all $n\in N$

g) $sin\theta+\sin 2\theta+\ldots+\sin(n\theta)=\frac{\sin\left(\frac{n+1}{2}\right)\theta.\sin\left(\frac{n\theta}{2}\right)}{\sin(\theta/2)}$ for all $n\in N$

h) $\int_0^{\pi/2}\cos^n x.\cos(nx)\,dx=\frac{\pi}{2^{n+1}}$ for all $n\in N.$

i) $\int_{0}^{\pi/2} \frac{\sin^2 nx}{sinx} dx = 1 + \frac{1}{3} + \frac{1}{5} + \ldots + \frac{1}{2n-1}$ for all $n \in N$.

j) $\sum_{r=0}^{n} r . n_{C_r} = n . 2^{n-1}$ for all $n \in N$.